

Fast and Memory-Efficient Algorithms for Quadratic Time–Frequency Distributions

A collection of M-files to compute time–frequency distributions (TFDs) from the quadratic class [1]. Memory and computational load is limited by controlling the level of over-sampling for the TFD. Oversampling in the TFD is proportional to signal length and bandwidth of the Doppler–lag kernel. Algorithms are optimised to four kernel types: nonseparable, separable, lag-independent, and Doppler-independent kernels.

Also included are algorithms to compute decimated, or sub-sampled, TFDs. Again, these algorithms are specific to the four kernel types but compute approximate TFDs by a process of decimation.

Requires Matlab or Octave (programming environments). Latest version available at http://otoolej.github.io/code/memeff_TFDs/.

1 Description

First, add paths using the `load_curdire` function:

```
1 >> load_curdire ;
```

There are two sets of TFD algorithms: one set computes oversampled TFDs and the other set computes decimated (sub-sampled or undersampled) TFDs. The first set, for computing oversampled TFDs, has four algorithms for specific kernel types, namely the

- non-separable,
- separable,
- Doppler-independent (DI),
- lag-independent (LI) kernels.

1.1 Oversampled TFDs

The function to generate these oversampled TFDs is `full_tfd.m`. The following examples, using a test signal, illustrates usage:

```
1 % generate test signal:
2 N=512;
3 x=gen_LFM(N,0.1,0.3) + gen_LFM(N,0.4,0.04);
4
5 % nonseparable kernel (Choi–Williams kernel):
```

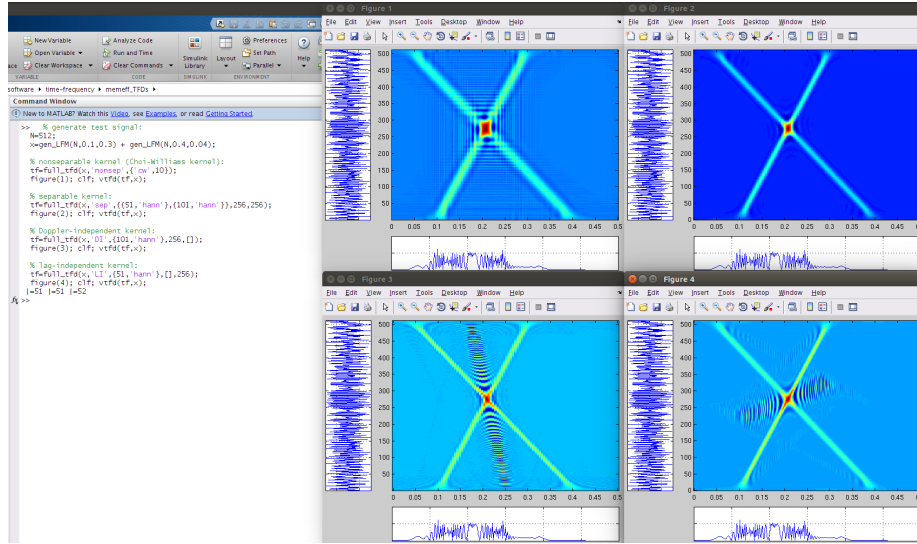


Figure 1: Examples of oversampled TFDs.

```

6 tf=full_tfd(x, 'nonsep', { 'cw', 10});
7 figure(1); clf; vtfd(tf,x);
8
9 % separable kernel:
10 tf=full_tfd(x, 'sep', {{51, 'hann'}, {101, 'hann'}} ,256,256);
11 figure(2); clf; vtfd(tf,x);
12
13 % Doppler-independent kernel:
14 tf=full_tfd(x, 'DI', {101, 'hann'}, 256, []);
15 figure(3); clf; vtfd(tf,x);
16
17 % lag-independent kernel:
18 tf=full_tfd(x, 'LI', {51, 'hann'}, [], 256);
19 figure(4); clf; vtfd(tf,x);

```

Type `help full_tfd` for more details.

1.2 Undersampled TFDs

Likewise, the algorithms for decimated TFDs are specific to the four kernel types. The function `dec_tfd` computes the decimated TFDs, as the following examples show:

```

1 N=1024; Ntime=64; Nfreq=128;
2 a=2; b=2;
3 ni=[100:2:900]; ki=[150:2:850];

```

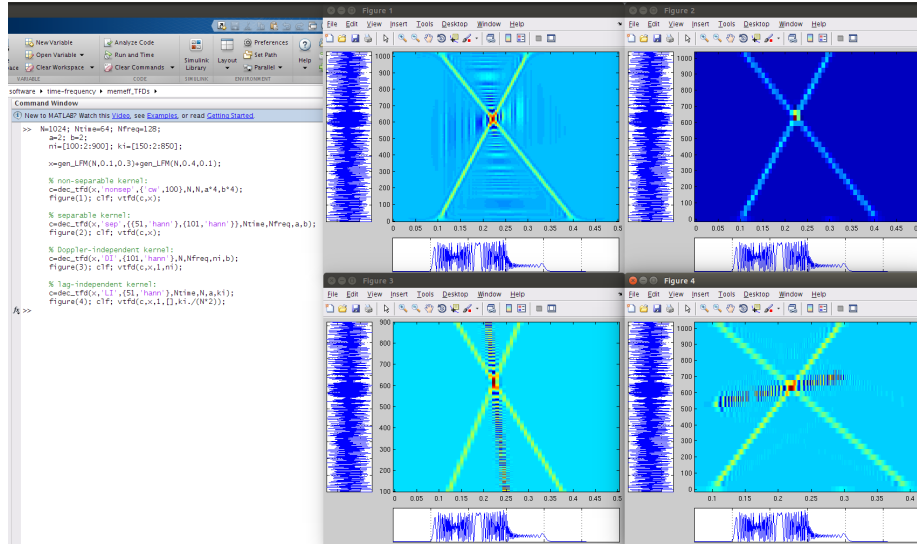


Figure 2: Examples of decimated TFDs.

```

4
5 x=gen.LFM(N,0.1,0.3) + gen.LFM(N,0.4,0.1);
6
7 % non-separable kernel:
8 c=dec_tfd(x,'nonsep',{ 'cw',100},N,N,a*4,b*4);
9 figure(1); clf; vtfd(c,x);
10
11 % separable kernel:
12 c=dec_tfd(x,'sep',{51,'hann'},{101,'hann'}),Ntime,Nfreq,
    a,b);
13 figure(2); clf; vtfd(c,x);
14
15 % Doppler-independent kernel:
16 c=dec_tfd(x,'DI',{101,'hann'},N,Nfreq,ni,b);
17 figure(3); clf; vtfd(c,x,1,ni);
18
19 % lag-independent kernel:
20 c=dec_tfd(x,'LI',{51,'hann'},Ntime,N,a,ki);
21 figure(4); clf; vtfd(c,x,1,[],ki./(N*2));

```

Type `help dec_tfd` for more details on this function.

| name | type | description |
|----------------|------|--|
| common | dir. | files to generate kernel functions |
| decimated_TFDs | dir. | files to generate decimated TFD for the 4 kernel types |
| full_TFDs | dir. | files to generate oversampled TFD for the 4 kernel types |
| utils | dir. | miscellaneous files |
| dec_tfd.m | file | computes decimated TFDs |
| full_tfd.m | file | computes oversampled TFDs |
| LICENCE.md | file | licence file |
| README.md | file | README file (markdown format) |
| README.pdf | file | this README file |
| load_curdir.m | file | adds paths for Matlab/Octave |

Table 1: Files and directories (dir.)

2 Files

All Matlab files (.m files) have a description and an example in the header. To read this header, type `help <filename.m>` in Matlab. Directory structure is in Table 1.

3 Requirements

Either Matlab (R2012 or newer, [Mathworks website](#)) or Octave (v3.6 or newer, [Octave website](#), with the ‘octave-signal’ add-on package).

4 Test Computer

- hardware: Intel(R) Xeon(R) CPU E5-1603 0 @ 2.80GHz; 8GB memory.
- operating system: Ubuntu GNU/Linux x86_64 distribution (Trusty Tahr, 14.04), with Linux kernel 3.13.0-27-generic
- software: Octave 3.8.1 (using Gnuplot 4.6 patchlevel 4), with ‘octave-signal’ toolbox and Matlab (R2009b, R2012a, and R2013a)

5 References

1. J.M. O’ Toole and B. Boashash, “Memory Efficient Algorithms for Quadratic TFDs”, Chapter 6.6; in *Time-Frequency Signal Processing and Analysis: A Comprehensive Reference*, Second Edition, Academic Press, pp. 374–385, 2016 (ISBN: 9780123984999).
2. J.M. O’ Toole and B. Boashash, “Fast and memory-efficient algorithms for computing quadratic time–frequency distributions”, *Applied and Computational Harmonic Analysis*, vol. 35, no. 2, pp. 350–358, 2013.

3. J.M. O Toole, M. Mesbah, and B. Boashash, “Improved discrete definition of quadratic time–frequency distributions,” *IEEE Transactions on Signal Processing*, vol. 58, Feb. 2010, pp. 906-911.
4. J.M. O’ Toole, M. Mesbah, and B. Boashash, “A New Discrete Analytic Signal for Reducing Aliasing in the Discrete Wigner-Ville Distribution”, *IEEE Transactions on Signal Processing*, vol. 56, no. 11, pp. 5427-5434, Nov. 2008.
5. J.M. O Toole, M. Mesbah, and B. Boashash, “Algorithms for discrete quadratic time–frequency distributions,” *WSEAS Transactions on Signal Processing*, vol. 4, May. 2008, pp. 320-329.

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